Variation in the nutrient composition of ingredients is common and unavoidable. The risk in achieving targeted nutrient levels with ingredients that vary in composition can be cost-effectively managed through the use of stochastic programming.

Stochastic Programming: Addressing Risk Due to Variability

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Variability and risk:

The primary objective of feed formulation is to generate a formula that meets the specifications set for it at the lowest possible cost. The inherent variability in nutrient composition of feed ingredients represents a major risk factor in formula optimization. The variation of protein, for example, is shown in Table 1 for some of the feed ingredients commonly used in the formulation of aquaculture feeds. The risk with nutrient variability in formulated diets is that the diets may not fully meet the nutrient requirements of the animal. Chick studies (Table 2) have demonstrated that increased variation of nutrients in diets can adversely affect weight gain and feed conversion (Duncan, 1988). It is conceivable that nutrient variation would also affect fish performance in a similar manner. The risk associated with nutrient variability must be handled diligently by feed manufacturers. While under-delivery of nutrients is detrimental to feed performance, over-formulating, i.e. specifying nutrients in excess of requirements is wasteful and expensive. Furthermore, reduction of variance to deliver consistent quality finished goods is expected in modern manufacturing industries.

Nott and Combs (1967) suggested that nutrient variability in the feed formulation process could be managed by providing a margin of safety for the nutrients. They recommended an adjustment of the nutrient means by subtracting 0.5 of the nutrient standard deviation (SD). For example, soybean meal with a mean protein value of 44.5% and an SD of 1.4 would have an adjusted protein value of 43.8% [(44.5)-(1.4÷2) = 43.8]. They proposed that these margins of safety values would then make up the ingredient database for the linear program.

However, nutrient adjustments (margins of safety) are not appropriate for a linear program. Technically, it is assumed that the input values (e.g., nutrient levels, animal requirements and ingredient cost) in a linear program are linear and are known with certainty. Because the variance of nutrients used in the formulation algorithm is the square of the standard deviation (i.e., variance = (standard deviation)^2), the formulation process becomes a nonlinear problem. Therefore, nutrient variation as a nonlinear input variable violates the assumptions of uncertainty and linearity for the linear program. The resulting consequence of a linear program formulation with an adjusted nutrient matrix is a costly over-formulation of the diet (Roush et al., 1996). A more appropriate diet formulation method is the use of stochastic programming (also referred to as chance-constrained programming).

***Table 1:** Mean and standard deviation of the crude protein content of some feed ingredients (After Burdett and Laws 1979).*

Ingredient	Number of Samples	Mean ± Standard Deviation
Corn	212	9.2 ± 0.5
Wheat	1002	11.3 ± 0.9
Soybean Meal	571	44.5 ± 1.4
Meat and Bone Meal		
Source a	27	46.5 ± 2.2
Source b	61	47.0 ± 2.0
Source c	99	46.5 ± 2.0
Source d	166	50.2 ± 2.6
Source e	140	46.0 ± 1.9
Herring Meal		
Source f	119	69.8 ± 2.2
Source g	42	69.2 ± 2.4

What is stochastic programming?

Stochastic programming is a nonlinear approach to feed formulation and it is a refinement in providing margins of safety for feed formulation. Stochastic refers to the variability of nutrients and the probability of meeting the nutrient requirement. Stochastic programming is, therefore, a method of feed formulation that can effectively incorporate nutrient variability into the formulation process to meet animal requirements with a measured level of certainty.

How does stochastic programming work?

To illustrate the differences in formulation, the three diets shown in Table 3 are formulated for a crude protein level of 32% with an energy level of 2800 kcal DE/kg. The numbers associated with the diet label represent the requested probability level. The diets are as follows:

- LP50 - an unadjusted linear program with a 50% probability of meeting or not meeting the nutrient requirement.
- LP69 - a margin of safety adjusted linear program with a 69% probability of meeting the nutrient requirement.
- SP69 - a stochastic program with a 69% probability of meeting the nutrient requirement.

For each of the diets, the target nutrient was protein at 32% of the diet. After computer formulation, the resulting level of probability was calculated for each of the diets.

The difference in the cost of the three rations is striking. The LP50 ration, which represents a linear program without margins of safety, had the lowest cost ($254.16 per ton). The LP69 diet formulated with a margin of safety produced a diet that had a cost of $256.22 per ton. The stochastic program (SP69) produced a diet with a $255.48 cost per ton. There is a $0.74 per ton difference in cost between the LP69 and SP69 formulations.

It should be noted, that the rations with a 69% probability are more costly than the 50% probability because there is an insurance cost with a nutrient margin of safety. The nutritional goal is to provide sufficient nutrient insurance without over-formulation.

A comparison of the calculated protein levels for the rations in Table 3 shows how the nutrient insurance is reflected in level of protein. That is, 32.00%, 32.66% and 32.42% protein for LP50, LP69 and SP69, respectively. The difference between LP69 and SP69 represents a 0.24% crude protein difference. The reason for this nutrient difference is reflected in the ability of the respective linear or stochastic program to accurately meet the requested 69% probability level. The LP69 calculated probability was 78%, while stochastic formulation (SP69) accurately provided the requested 69% probability. The bottom line is that the linear program (LP69) over-formulated the diet by 9%, while the stochastic program (SP69) accurately met the requested 69% probability

Although the difference between the protein levels and probabilities appears to be minimal, it must be remembered that the differences and the economic consequences of over-formulated levels of nutrients are real. In the formulation process, each ingredient is accurately measured and included in the mixture according to the percentage indicated by the recipe.

Table 2: *Experimental chick responses to dietary protein variation (Duncan, 1988)*[1]

Treatment	Gain (g)	Feed Conversion
Control	773a	1.74a
10 % C.V.	716b	1.82b
20% C.V.	703b	1.86c

[1]Values in a column with the same letter are not significantly different ($p<0.05$)

Table 3: *Comparison of Catfish diets formulated by a linear program (LP50), a linear program with a margin of safety (LP69) and a stochastic program (SP69).*

Ingredients	LP50	LP69	SP69
	------------------	%	------------------
Corn	34.61	33.17	33.69
Wheat	5.00	5.00	5.00
Soybean Meal (49.5)	50.31	51.94	51.35
Fish meal, Menhaden	7.00	7.00	7.00
Fish Oil	1.28	1.09	1.16
Calcium Phosphate	1.50	1.50	1.50
Vitamin Mix	0.25	0.25	0.25
Mineral Mix	0.05	0.05	0.05
Cost, $/ton	254.16	256.22	255.48
Printout Protein, %	32.00	32.00	32.00
Calculated Protein, %	32.00	32.66	32.42
Requested Probability, %	50.00	69.00	69.00
Calculated Probability,%	50.00	78.00	69.00

How to use stochastic programming in practical Formulation?

Formulators who plan to use stochastic programming must have an estimate of nutrient means and standard deviations representative of the ingredients being included in the diet formulation. Collection of data on nutrient means and standard deviations at most feed mills is a retrospective activity. By the time the nutrient analysis is available on a batch of ingredients, the ingredients probably have been mixed in a diet and fed to the animals. Developments in Near Infrared Spectroscopy (NIRS) technology are making the collection of statistical data on ingredients available on a real time basis[1]. However, it must be remembered that NIRS is a sampling procedure for feed ingredients, the results of which will have means and standard deviations associated with them. These nutrient mean and standard deviation values can be incorporated in a stochastic program.

In addition to mean and standard deviation values, the diet formulator requires software that can compute a stochastic formula. The linear and stochastic software program used to formulate these rations was Pennfeed© which was developed at Penn State University (Zhang, 1999). Two companies have been reported to currently have commercial stochastic programs available for formulation: Creative Formulation Concepts (formerly Agridata Systems, Inc.) and Format International. Further development and availability of stochastic software will be determined by customer demand.

In conclusion, nutrient variability in ingredients presents an element of risk to diet formulation as well as animal performance. Any attempt to adjust a linear program for nutrient variability results in costly over-formulation of the ration. A stochastic program, on the other hand, provides a feed formulation method that can deal with nutrient variability to accurately meet preset levels of risk.

References

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[1]Editor's Note: See *Aqua Feeds: Formulation & Beyond*, Vol. 1, Issue 2, pages 28-30 for an article on the application of NIRS in feed quality management,

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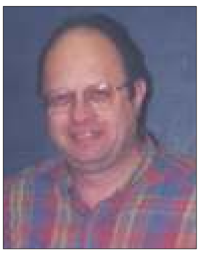

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